Controllability of scalar wave-type equations and of coupled systems of PDEs by a reduced number of controls

Fatiha ALABAU-BOUSSOUIRA*

8^{th} Euro-Maghrebian Workshop on Evolution Equations, Lecce June 11^{th}-15^{th} 2012, Italy

May 15, 2012

Abstract

The first parts of the course will be devoted to the control of scalar wave-type equations and their abstract version. We will recall the abstract setting in the semi-group framework and the dual notions of observability and controllability, through the Hilbert Uniqueness Method. We will also present some of the tools to prove the admissibility and the observability for such equations. The second part of the course will present some motivations and recent results on controllability/observability for coupled hyperbolic systems by a reduced number of controls/observations. This situation occurs whenever the number of controls is strictly less than the number of unknowns (or equations) of the coupled system of PDE's. The main goal is to control all the components of the state-vector, even though some of them are not directly controlled. Similar questions occur for the null-controllability of parabolic (resp. Schrödinger) coupled systems. We consider localized as well as boundary controls, and localized couplings. In all these cases, one of the main challenging question is to be able to control the full system by controls such that the control region do not meet the region where the coupling is localized. We will present several positive results in this direction for hyperbolic systems and further give applications to null controllability for parabolic and Schrödinger coupled systems by a reduced number of controls.

References

[1] F. Alabau-Boussouira, *Indirect boundary observability of a weakly coupled wave system.* C. R. Acad. Sci. Paris, t. 333, Série I, 645–650 (2001).

*Université de Lorraine and Inria, 57045 METZ Cedex 1 (France)

[2] F. Alabau-Boussouira, *A two-level energy method for indirect boundary observability and controllability of weakly coupled hyperbolic systems.* Siam J. Control Opt. 42, 871–906 (2003).

[3] F. Alabau-Boussouira and M. Léautaud, *Indirect controllability of locally coupled systems under geometric conditions.* C. R. Acad. Sci. Paris, t. 349, Série I, 395–400 (2011).

[4] F. Alabau-Boussouira and M. Léautaud, *Indirect controllability of locally coupled wave-type systems and applications.* Accepted for publication in Journal de Mathématiques Pures et Appliquées.

[5] F. Alabau-Boussouira, *Controllability of cascade coupled systems of multi-dimensional evolution PDE's by a reduced number of controls.* arXiv:1109.6645v2 (2011).

[6] F. Ammar-Khodja and A. Benabdallah and C. Dupaix, *Null controllability of some reaction-diffusion systems with one control force.* J. Math. Anal. Appl. 320, 928–943 (2006).

[7] F. Ammar-Khodja and A. Benabdallah and C. Dupaix and M. González-Burgos, *A Kalman rank condition for the localized distributed controllability of a class of linear parabolic systems.* J. Evol. Equations 9, 267–291 (2009).

[8] F. Ammar-Khodja, A. Benabdallah, M. González-Burgos, L. de Teresa, *Recent results on the controllability of linear coupled parabolic problems: a survey.* Mathematical Control and Related Fields 1, 267–306 (2011).

[9] J.-M. Coron, *Control and nonlinearity.* Mathematical Surveys and Monographs, 136. American Mathematical Society, Providence, RI, (2007).

[10] J.-M. Coron, S. Guerrero, L. Rosier, *Null controllability of a parabolic system with a cubic coupling term.* SIAM J. Control Optim. 48, 5629–5653 (2010).

[11] S. Ervedoza and E. Zuazua, *Observability of heat processes by transmutation without geometric restrictions.* Math. Control Relat. Fields 1, 177-187 (2011).

[12] E. Fernández-Cara and M. González-Burgos and L. de Teresa, *Boundary controllability of parabolic coupled equations.* J. Functional Anal. 259, 1720–1758 (2010).

[13] M. González-Burgos and L. de Teresa, *Controllability results for cascade systems of m coupled parabolic PDEs by one control force.* Port. Math. 67, 91–113 (2010).

[14] O. Kavian and L. de Teresa, *Unique continuation principle for systems of parabolic equations.* ESAIM COCV 16, 247–274 (2010).

[15] V. Komornik, *Exact Controllability and Stabilization. The Multiplier Method.* Collection RMA, vol. 36, Masson–John Wiley, Paris–Chicester (1994).

[16] I. Lasiecka, *Mathematical control theory of coupled PDEs.* CBMS-NSF Regional Conference Series in Applied Mathematics, 75. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, (2002).

[17] M. Léautaud, *Spectral inequalities for non-selfadjoint elliptic operators and application to the null-controllability of parabolic systems.* J. Funct. Anal. 258, 2739–2778 (2010).

[18] M. Léautaud, *Quelques problèmes de contrôle déquations aux dérivées partielles : inégalités spectrales, systèmes couplés et limites singulières.* Thèse de doctorat de l'université de Paris 6 (2011).

[19] J. L. Lions, *Contrôlabilité exacte et stabilisation de systèmes distribués.* Vol. 1–2, Masson, Paris (1988).

[20] L. Miller, *The control transmutation method and the cost of fast controls.* Siam J. Cont. Opt. 45, 762–772 (2006).

[21] G. Olive, *Null-controllability for some linear parabolic systems with controls acting on different parts of the domain and its boundary.* Mathematics of Control, Signals and Systems 23, 257–280 (2012).

[22] K.-D. Phung, *Observability and control of Schrödinger equations.* Siam J. Cont. Opt. 40, 211–230 (2001).

[23] L. Rosier and L. de Teresa, *Exact controllability of a cascade system of conservative equations.* C. R. Acad. Sci. Paris, Ser. I 349, 291–296 (2011).

[24] T. Seidman, *Two results on exact boundary control of parabolic equations.* Appl. Math. Optim. 11, 145–152 (1984).