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On a nonlinear flux–limited equation arising in the transport of morphogens

Motivated by a mathematical model for the transport of morphogenes in biological systems, we study existence and uniqueness of entropy solutions for the mixed initial-boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = \left(\mathbf{a}(u, u_x)\right)_x, & \text{in} \quad]0,T[\times]0,L[, \\ -\mathbf{a}(u(t,0), u_x(t,0)) = \beta > 0 \;\text{ and }\; u(t,L) = 0, & \text{on} \quad t \in]0,T[, \\ u(0,x) = u_0(x), & \text{in} \quad x \in]0,L[, \end{cases} \tag{1}$$

being

$$\mathbf{a}(z,\xi) := \nu \frac{|z|\xi}{\sqrt{z^2 + \frac{\nu^2}{c^2}|\xi|^2}}.$$

The equation in (1) was introduced by Ph. Rosenau (1992) to correct the infinite speed of propagation of the classical diffusion equation.

In this lecture we also study some qualitative properties of the entropy solutions of (1), questions related to the existence of steady states, the finite speed of propagating fronts or the asymptotic behaviour of the solutions.

[1]Joint works with F. Andreu, J. Calvo, J. Soler and M. Verbeni