Abdelaziz Rhandi

Invariant measures: Existence, Uniqueness and global properties

The aim of the lectures is to give the state of the art of invariant measures for second order differential operators.

We present briefly the notion of invariant measures for dynamical systems and its relationship with the asymptotic behaviour of the solution to ordinary differential equations. For the existence of invariant measures, we recall the Krylov-Bogoliubov theorem.

More time will be dedicated to invariant measures associated with second order differential operators in $\mathbf{R}^N$. We propose here to study global regularity properties of invariant measures and present different examples.

In particular, under suitable conditions, we prove global boundedness of the density, Sobolev regularity, a Harnack inequality and pointwise upper and lower bounds. The proofs rely upon Lyapunov functions and Moser's iteration techniques.

The lectures are based on recent works by V.I. Bogachev, G. Metafune, N.V. Krylov, D. Pallara, A. Rhandi and S.V. Shaposhnikov.