# Program of the 8th Euro-Maghrebian workshop on Evolution Equations

Dipartimento di Matematica e Fisica "Ennio De Giorgi" – 11–15 June, 2012

**Scientific Committee:** Aissa Aibeche (Sétif), Wolfgang Arendt (Ulm), Mohammad Ali Jendoubi (Bizerte), Alessandra Lunardi (Parma), Lahcen Maniar (Marrakech), José Mazon (Valencia), El Maati Ouhabaz (Bordeaux), Roland Schnaubelt (Karlsruhe)

**Sponsors:** Università del Salento, Karlsruhe Institute of Technology, GNAMPA – INdAM, Monte dei Paschi di Siena

## List of Minicourses

**Fatiha Alabau-Boussouira (Metz)**: *Controllability of scalar wave-type equations and of coupled systems of PDEs by a reduced number of controls*

**Vicent Caselles (Barcellona)**: *Variational Models in Image Processing*

**Abdelaziz Rhandi (Salerno)**: *Invariant measures: Existence, Uniqueness and global properties*

## List of invited talks

**Ralph Chill (Dresden)**: *Dirichlet and Neumann boundary conditions for the p-Laplace operator: What is in between?*

**Giovanni Galdi (Pittsburgh)**: *On the Steady Motion of a Coupled System Solid-Liquid*

**Jerry Goldstein (Memphis)**: *Energy aspects of strongly damped wave equations*

**Gisèle Ruiz Goldstein (Memphis)**: *The functional analysis behind global wellposedness for reaction-diffusion systems having boundary diffusion and bounded total mass but finite time blowup*

**Luca Lorenzi (Parma)**: *An overview on nonautonomous Kolmogorov equations*

**José Mazon (Valencia)**: *On a nonlinear flux-limited equation arising in the transport of morphogens*

**Sylvie Monniaux (Marseille)**: *The divergence theorem involving the pointwise non-tangential trace*

**Serge Nicaise (Valenciennes)**: *Uniformly exponentially or polynomially stable approximations for second-order evolution equations*

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**M. Boukdir ()**: *Characterization of a polinomially bounded semigroup in term of the resolvent*

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**Derrardjia Ishak (El Tarf)**: *Stability in nonlinear neutral differential equations with variable delays using fixed point theory*

**El Azzouzi Mohamed (Marrakech)**: *Approximate Positive Controllability for Retarded Functional Equations*

**Fackler Stephan (Ulm)**: *Regularity of Semigroups via the Asymptotic Behaviour at Zero*

**Manzo Rosanna (Salerno)**: *Optimization of traffic flows using a fluid dynamic model*

**Mourou Sami (Tunis)**: *Kernel Estimates for Generalized Ornstein-Uhlenbeck Operators*

**Spina Chiara (Lecce)**: *Elliptic operators with unbounded diffusion coefficients*

# Book of abstracts
## Minicourses

Fatiha Alabau-Boussouira

**Controllability of scalar wave-type equations and coupled systems of PDEs by a reduced number of controls**

The first parts of the course will be devoted to the control of scalar wave-type equations and their abstract version. We will recall the abstract setting in the semi-group framework and the dual notions of observability and controllability, through the Hilbert Uniqueness Method. We will also present some of the tools to prove the admissibility and the observability for such equations. The second part of the course will present some motivations and recent results on controllability/observability for coupled hyperbolic systems by a reduced number of controls/observations. This situation occurs whenever the number of controls is strictly less than the number of unknowns (or equations) of the coupled system of PDE's. The main goal is to control all the components of the state-vector, even though some of them are not directly controlled. Similar questions occur for the null-controllability of parabolic (resp. Schrödinger) coupled systems. We consider localized as well as boundary controls, and localized couplings. In all these cases, one of the main challenging question is to be able to control the full system by controls such that the control region do not meet the region where the coupling is localized. We will present several positive results in this direction for hyperbolic systems and further give applications to null controllability for parabolic and Schrödinger coupled systems by a reduced number of controls.

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Vicent Caselles

**Variational Models in Image Processing**

The purpose of this series of lectures is to discuss some variational models for different image processing tasks. Specifically, we will cover the following topics:

1. An introduction to total variation models in image processing, covering the restoration problem and optical flow computation.
2. We will consider variational formulation for exemplar based image inpainting and discuss its qualitative properties.
3. We will discuss a variational model for video editing, that is, for object insertion and propagation along the video sequence.
4. Finally, we will discuss a variational model for contrast enhancement of color images.

Abdelaziz Rhandi

**Invariant measures: Existence, Uniqueness and global properties**

The aim of the lectures is to give the state of the art of invariant measures for second order differential operators.
We present briefly the notion of invariant measures for dynamical systems and its relationship with the asymptotic behaviour of the solution to ordinary differential equations. For the existence of invariant measures, we recall the Krylov-Bogoliubov theorem.
More time will be dedicated to invariant measures associated with second order differential operators in $\mathbf{R}^N$. We propose here to study global regularity properties of invariant measures and present different examples. In particular, under suitable conditions, we prove global boundedness of the density, Sobolev regularity, a Harnack inequality and pointwise upper and lower bounds. The proofs rely upon Lyapunov functions and Moser's iteration techniques.
The lectures are based on recent works by V.I. Bogachev, G. Metafune, N.V. Krylov, D. Pallara, A. Rhandi and S.V. Shaposhnikov.

## List of invited talks

Ralph Chill

**Dirichlet and Neumann boundary conditions for the $p$-Laplace operator: What is in between?**

We characterize all order preserving semigroups on $L^2(\Omega)$ which are generated by convex, lower semicontinuous, local functionals and which are sandwiched between the semigroups generated by the $p$-Laplace operator with Dirichlet and Neumann boundary conditions; here, $p \in (1, \infty)$ and $\Omega \subseteq \mathbf{R}^N$ is a bounded domain with Lipschitz continuous boundary. We show that every such semigroup is generated by the $p$-Laplace operator with Robin type boundary conditions.
This is joint work with M. Warma.

Giovanni P. Galdi

**On the Steady Motion of a Coupled System Solid-Liquid**

The main topic of this talk is centered around the unconstrained (free) motion of an elastic solid, $\mathcal{B}$, in a Navier-Stokes liquid, $\mathcal{L}$, occupying the whole space outside $\mathcal{B}$, under the assumption that a constant body force is acting on $\mathcal{B}$. More specically, we are interested in the steady motion of the coupled system $\{\mathcal{B}, L\}$, which means that there exists a frame with respect to which the relevant governing equation possess a time-independent solution. We discuss the existence of such a frame and of corresponding steady solutions, and show that they actually exist, provided some smallness restrictions are imposed on the physical parameters, and the reference conguration of $\mathcal{B}$ satises suitable geometric properties.
Part of this work is in collaboration with Josef Bemelmans (RWTH Aachen) and Mads Keyd (TU Darmstadt).

Jerry Goldstein

**Energy aspects of strongly damped wave equations**

Of concern are the asymptotics for

$$u(t) + Bu(t) + Au(t) = 0, \qquad t > 0$$

where $A, B$ are commuting positive selfadjoint operators on a complex Hilbert space $H$ with $A$ unbounded. The usual (abstract) wave equation is when $B = 0$. The case of $B = bI$ with $b$ a positive constant refers to the telegraph equation. Strong damping means that $B$ is unbounded (and $B$ is smaller than $A$ in some sense). Of concern are several types of results, including asymptotic equipartition of energy, when the ratio of kinetic to potential energy has limit 1 as $t$ goes to infinity. Other issues include overdamping and asymptotic parabolicity.
My collaborators on the recent results in these areas are Ted Clarke, Genni Fragnelli, Gisèle Goldstein, Gustavo Perla, Guillermo Reyes, and Silvia Romanelli.

Gisèle Ruiz Goldstein

**The functional analysis behind global wellposedness for reaction-diffusion systems having boundary diffusion and bounded total mass but finite time blowup**

We consider reaction-diffusion systems of the form

$$\frac{\partial u_i}{\partial t} = d_i \Delta u_i + F_i(u_1, ..., u_n)$$

for $i = 1, ..., n$. This system can be written in the abstract form

$$du/dt = Au + B(u)$$

where the partial differential equation $du/dt = Au$ models the diffusion and the ordintary differential equation $dw/dt = B(w)$ models the chemical reactions. This simple model has been amazingly effective in describing physical phenomena observed in problems from science and engineering. Some recent applications in chemical engineering required dynamic boundary conditions and diffusion along the spatial boundary in order to accurately model the physical situation. In many of these applications $u_i(x,t)$ represents a mass density for one of the components, and

$$\|u(\cdot,t)\|_{L^1} = \sum_i \|u_i(\cdot,t)\|_1$$

is bounded independently of the time; yet the quantity

$$\|u(\cdot,t)\|_\infty$$

blows up in finite time. A natural question is: Does the (local) solution extend to a global in time weak solution? Sometimes yes and sometimes no. The proof of global wellposedness can be reduced to showing that the corresponding linear diffusion semigroup (corresponding to the operator $A$) is compact on the appropriate $L^1$-type space. In these problems the appropriate $L^1$-type space is the space $X_1$ introduced by Favini-Goldstein-Goldstein-Romanelli; $X_1$ requires appropriate weights in the boundary integral portion of the norm. In this talk we will explain the solution of this problem in a quite general context. This problem is of independent interest since it solves a problem of Agmon, Douglis and Nirenberg from 1959.
The work presented is joint with Jerry Goldstein (University of Memphis) and Michel Pierre (École Normale Supérieure de Cachan).

Luca Lorenzi

**An overview on nonautonomous Kolmogorov equations**

In this talk we report on some recent results for evolution operators associated with nondegenerate nonautonomous elliptic operators with unbounded coefficients defined in $I \times \mathbf{R}^N$ where $I$ is a right-halfline.

José Mazon

**On a nonlinear flux-limited equation arising in the transport of morphogens**

Motivated by a mathematical model for the transport of morphogenes in biological systems, we study existence and uniqueness of entropy solutions for the mixed initial-boundary value problem

$$\begin{cases} \frac{\partial u}{\partial t} = (\mathbf{a}(u, u_x))_x, & \text{in} \quad ]0,T[\times]0,L[, \\ -\mathbf{a}(u(t,0), u_x(t,0)) = \beta > 0 \quad \text{and} \quad u(t,L) = 0, & \text{on} \quad t \in ]0,T[, \\ u(0,x) = u_0(x), & \text{in} \quad x \in ]0,L[, \end{cases} \tag{1}$$

being

$$\mathbf{a}(z,\xi) := \nu \frac{|z|\xi}{\sqrt{z^2 + \frac{\nu^2}{c^2}|\xi|^2}}.$$

The equation (1) was introduced by Ph. Rosenau (1992) to correct the infinite speed of propagation of the classical diffusion equation. In this lecture we also study some qualitative properties of the entropy solu-

tions of (1), questions related to the existence of steady states, the finite speed of propagating fronts or the asymptotic behaviour of the solutions.

Sylvie Monniaux

**The divergence theorem involving the pointwise non-tangential trace**

De Giorgi-Federer produced a Divergence Theorem in an optimal class of domains $\Omega \subseteq \mathbb{R}^n$, domains of locally finite perimeter, for regular vector fields $\vec{F}$ with Lipschitz components. They proved that there exists a unique measure theoretic unit outer normal $\nu_\Omega$ such that

$$\int_\Omega \operatorname{div} \vec{F}\, d\mathcal{L}^n = \int_{\partial_*\Omega} \vec{F} \cdot \nu_\Omega \, d\mathcal{H}^{n-1},$$

where $\mathcal{L}^n$ stands for the $n$-dimensional Lebesgue measure, $\mathcal{H}^{n-1}$ the $n-1$-dimensional Hausdorff measure in $\mathbb{R}^n$, $\partial_*\Omega$ the measure theoretic boundary of $\Omega$. In this talk, this result is extended to vector fields in $L^1_{\mathrm{loc}}(\Omega;\mathbb{R}^n)$ whose divergence belongs to $L^1(\Omega)$ and its non-tangential maximal function belongs to $L^1(\partial\Omega)$. This is a joint work with Dorina Mitrea, Irina Mitrea and Marius Mitrea.

Serge Nicaise

**Uniformly exponentially or polynomially stable approximations for second order evolution equations**

We will consider the approximation of second order evolution equations. It is well known that the approximated system by finite element or finite difference is not uniformly exponentially or polynomially stable with respect to the discretization parameter, even if the continuous system has this property. Our goal is to damp the spurious high frequency modes by introducing numerical viscosity terms in the approximation scheme. With these viscosity terms, we show the exponential or polynomial decay of the discrete scheme when the continuous problem has such a decay and when the spectrum of the spatial operator associated with the undamped problem satisfies the generalized gap condition. Some illustrative examples will be also presented.
This talk is based on a joint work with F. Abdallah (Université de Valenciennes et du Hainaut Cambrésis, France and Université Libanaise, Beyrouth, Liban), J. Valein (Nancy-Université & INRIA (Project-Team CORIDA), France) and A. Wehbe (Université Libanaise, Beyrouth, Liban).

Enrico Priola

**Global Lipschitz regularizing effects for parabolic equations with singular coefficients**

We prove global bounds on the spatial gradient of viscosity solutions to second order linear and nonlinear parabolic Cauchy problems in $(0,T) \times R^N$. Our assumptions include the case that the coefficients be both unbounded and with very mild local regularity (possibly weaker than the Dini continuity), the estimates only depending on the parabolicity constant and on the modulus of continuity of coefficients (but not on their $L^\infty$-norm). Our proof provides the analytic counterpart to the probabilistic proof used in E. Priola and F. Y. Wang (J. Funct. Anal. 2006) to get this type of gradient estimates in the linear case. We actually extend such estimates to the case of possibly unbounded data and solutions as well as to the case of nonlinear operators of Bellman-Isaacs type. We investigate both the classical regularizing effect (at time $t > 0$) and the possible conservation of Lipschitz regularity from $t = 0$, and similarly we prove global Hölder estimates under weaker assumptions on the coefficients. The estimates we prove for unbounded data and solutions

seem to be new even in the classical case of linear equations with bounded and Hölder continuous coefficients. Applications to Liouville type theorems are also given.
This is a joint work with A. Porretta (Roma "Tor Vergata").

## List of communications

Davide Addona

**Schauder estimates for nonautonomous Ornstein-Uhlenbeck operator in spaces of continuous weighted functions**

In my work, I study the non Homogenous backward Cauchy problem associated to the nonautonomous Ornstein-Uhlenbeck operator in the space of weighted functions. The main tools to get Schauder estimates will be the representation formula of solution to the homogenous Cauchy problem, and interpolation arguments.

Luciana Angiuli

**Summability improving in nonautonomous Kolmogorov equations**

The main purpose of this talk is to show how the occurrence of some functional inequalities is related to the smoothing effects of the evolution operators associated with nonautonomous Kolmogorov equations.

Waed Dada

**A new view at higher rank numerical ranges of Hilbert space operators**

The study of the numerical range has a long history and there is much current research on this concept and its generalization. In the first part of the talk, the classical numerical range of a bounded operator on a Hilbert space is explained and the quadratic numerical range $W^2(\mathcal{A})$ is presented where $\mathcal{A}$ is block operator matrix. Then we introduce the rank-two numerical range. We discuss its properties and the relation between the three concepts of numerical range. Moreover we generalize it to numerical ranges of higher rank.

Ishak Derrardjia

**Stability in nonlinear neutral differential equations with variable delays using fixed point theory**

The purpose of this paper is to use a fixed point approach to obtain asymptotic stability results of a nonlinear neutral differential equation with variable delays. An asymptotic stability theorem with a necessary and sufficient condition is proved. In our consideration we allow the coefficient functions to change sign and do not require bounded delays. The obtained results improve and generalize those due to Burton, Zhang and Raffoul. We end by giving three examples to illustrate our work.

Mohamed El Azzouzi

**Approximate Positive Controllability For Boundary Control Systems**

This presentation is devoted to study the Approximate positive controllability for boundary control systems. Approximate positive controllability of such systems is characterized. The abstract results are applied to retarded functional equations.

Stephan Fackler

**Regularity of Semigroups via the Asymptotic Behaviour at Zero**

An interesting result by T. Kato and A. Pazy says that a contractive semigroup $(T(t))$ on a uniformly convex space $X$ is holomorphic iff $\limsup_{t\downarrow 0} ||T(t) - Id|| < 2$. We study extensions of this result which are valid on arbitrary Banach spaces for semigroups which are not necessarily contractive. This allows us to prove a general extrapolation result for holomorphy of semigroups on interpolation spaces of exponent $\theta$ in $(0, 1)$.

Rosanna Manzo

**Optimization of traffic flows using a fluid dynamic model**

The aim of this work is to present some recent results on modeling and optimization of road traffic networks by a fluid dynamic approach. Road networks consist of a finite set of roads, that meet at some junctions. The dynamics is governed on each road by a conservation law. In order to uniquely solve the Riemann Problem at junctions and to construct solutions via Wave Front Tracking, as the system is under-determined even after imposing the conservation of cars, the following assumptions are made: the incoming traffic distributes to outgoing roads according to fixed (statistical) distribution coefficients; drivers behave in order to maximize the through flux. More precisely, if the number of incoming roads is greater than that of outgoing roads, one has also to introduce right of way parameters. Some cost functionals have been defined and maximized or minimized to analyze and improve the traffic behavior using distribution coefficients and right of way parameters as controls: J1 measuring car average velocity, J2 the average traveling time, J3 the total flux of cars, J4 the car density, J5, the Stop and Go Waves functional (SGW), the velocity variation, J6 the kinetic energy, and finally J7 measuring the average traveling time weighted with the number of cars moving on each road. Recently the problem of traffic redirection in the case an accident occurs in a congested area has been considered. We focus on simple junctions with two incoming roads and two outgoing ones and define a cost functional that measures the asymptotic average velocity of emergency vehicles. Fixing an incoming and an outgoing road for the emergency vehicle, we determine the local distribution coefficients which maximize such functional at a single junction. Then we use the local optimal coefficients at each node of the network, following a decentralized approach. All analytical result are tested by simulation, performed both for simple junctions or cascade networks, evaluating global performances of local optimal parameters on the network.

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Sami Mourou

**Kernel Estimates for Generalized Ornstein-Uhlenbeck Operators**

Recently, Metafune, Prüss, Rhandi and Schnaubelt [1] proved that the generalized Ornstein-Uhlenbeck operator

$$A_{\Phi,G}u = \Delta u - \nabla\Phi.\nabla u + G.\nabla u$$

with domain $W^{2,p}(\mathbb{R}^N,\mu)$ generates an analytic semigroup $T$ on $L^p(\mathbb{R}^N,\mu)$, $1 < p < \infty$, where $\mu(dx) = e^{-\Phi(x)}dx$. In this work we prove that the semigroup $T$ is a semigroup of integral operators given by

$$(T(t)u)(x) = \int_{\mathbb{R}^N} k_t(x,y)e^{\frac{1}{2}(\Phi(x)-\Phi(y))}u(y)dy, \text{ a.e. } x \in \mathbb{R}^N,\ u \in L^p(\mathbb{R}^N,\mu),\ t > 0,$$

where the kernel $k_t \in L^\infty(\mathbb{R}^N \times \mathbb{R}^N)$ satisfies Gaussian estimates.

**References**

[1] G. Metafune, J. Prüss, A. Rhandi, R. Schnaubelt, *$L^p$-regularity for elliptic operators with unbounded coefficients*. Adv. Diff. Equ. n.10 1131-1164 (2005).

Chiara Spina

**Elliptic operators with unbounded diffusion coefficients**

(Joint work whit Giorgio Metafune). We prove that, for $N \geq 3$, $\alpha > 2$, $\frac{N}{N-2} < p < \infty$, the operator $Lu = (1+|x|^\alpha)\Delta u$ generates an analytic semigroup in $L^p$ which is contractive if and only if $p \geq \frac{N+\alpha-2}{N-2}$. For $\alpha < \frac{N}{p'}$, we provide an explicit description of the domain. Spectral properties of the operator $L$ are also obtained. Moreover, by the equivalence between ultracontractivity and weighted Nash inequalities, we deduce some pointwise estimates for the parabolic kernel associated with the operator $L$. Similar results remain true for more general operators.